Dance Our Way to Geometric

Students slide into reflections, rotations, and translations and take giant leaps forward in their geometric understanding.

ing Transformations

Alison E. Leonard and
Nicole A. Bannister

Telling students that a 1 degree angle is 1/360 of a circle is unlikely to help them develop the type of spatial reasoning necessary for drawing a 45 degree angle. However, if students embody degree measure as an amount of turn by stretching out their arms and twisting their torsos to experience, practice, and perform different degree measures, then they will not only have a better spatial sense about what 45 degrees means but also be better positioned to create angles precisely with tools like protractors and dynamic geometry software.

Research suggests that embodying mathematical concepts through movement supports improvements in students' spatial fluency, communication, collaboration, critical thinking, and problem-solving skills (Alibali and Nathan 2012; Cone and Cone 2011; Sarama and Clements 2009). This article describes a dance-based strategy that middle school mathematics teachers can use to leverage embodied movement in their instructional repertoires. We take cues from dance educators for imagining this work because they have disciplinary expertise in the practice of movement as a form of inquiry. They define dance broadly as purposeful sequences of movement and stillness that communicate meaning (Leonard 2017; Hanna 2015). This means that we all potentially use dance, consume dance, and participate in dance every day, from the sequences of movements and stillness we use in our morning routines, like brushing our teeth, to the purposeful gestures we use when communicating, like using an arm to represent the slope of a line. These become dances when we view such movements as aesthetic and meaningful beyond their utilitarian purposes. Teachers and students can take comfort in knowing that these pedestrian sequences of gestures count just as much as technical ballet moves in dancing, which busts both the myth that a person must be trained in dance to be a dancer and the myth that dance is not an appropriate classroom strategy.

Early transformational geometry serves as a natural curricular access point for using dance to embody mathematical ideas because students are expected to use reflections, rotations, and translations to make meaning of bigger ideas about congruence and similarity by the end of eighth grade (CCSSI 2010; Driscoll et al. 2007; Seago et al. 2014). The Common Core State Standards for Mathematics (CCSSM) prioritizes a transformational approach, evidenced by core content standards oriented toward movement (CCSSI 2010). Middle school teachers who prefer a traditional approach, which is instead organized around set and proof (Usiskin 1972), may be unfamiliar with a transformations-based perspective (McCallum 2011; Seago et al. 2014). In addition, students may be challenged by the spatial reasoning skills employed by a movement-oriented approach (Bruce and Hawes 2015). An embodied learning strategy is appropriate for this work, given the need for pedagogies with increased potential for supporting middle school students' learning of these conceptual content challenges at a time when they are experiencing a convergence of overwhelming physiological, social, and intellectual changes (Caskey and Anfara 2014). In response, we designed the Transformations Dance performance task as a way to engage students in transformational geometry in ways that meet their content and developmental needs, as well as prepare them for future problem-solving tasks involving reflections, rotations, and translations.

THE TRANSFORMATIONS DANCE

The Transformations Dance was designed to introduce middle school students to translations, rotations, and reflections through embodied exploration, dance composition, and performance. It originated in and continues to be used in an arts-in-education course for preservice teachers, as well as in a preservice teacher geometry course and an in-service middle school mathematics teacher professional development context. Although the prospect of using dance-based pedagogies in schools may feel like an uncomfortable, daunting charge for many teachers (Leonard and Odutola 2016; Kaufmann and Ellis 2007), our collective successes with prospective and practicing teachers have aligned with known benefits of learning mathematics through dance—as have teachers' reports of classroom implementation with middle school students—which suggests that taking the risk is worth it. Embodied explorations and performances of transformations stand likely to provide essential content experiences for students at a critical time of early adolescent development, thereby inspiring the following fully adaptable implementation guide for middle school teachers interested in movement as inquiry.

PERFORMANCE TASK OVERVIEW

The Transformations Dance provides opportunities for improving and assessing students' conceptual understanding of isometric transformations and spatial reasoning through

whole- and small-group tasks involving movement-based inquiry, discussion, decision making, and choreography. Essentially, student groups embody a variety of isometric transformations, sequence them together into a choreographed dance, and perform their compositions for the class. In the next sections, we provide details for the five subtasks that we used to support students' progression through the Transformations Dance—Warm-up, Exploration, Composition, Sharing, and Reflection—which together take about one or two classroom sessions to complete, depending on context. We describe the modest amount of materials that teachers need to prepare in advance and propose implementation strategies that invite participation by students with varying comfort levels, physical needs, background experiences, and math histories.

An open floor space best supports student engagement during this movement inquiry, so we suggest pushing desks to the outer walls of the classroom, relocating to a larger space in the school, or going outside. However, one could modify the activity to take place with students using the space around their desks or using hand gestures while seated in table groups.

Fig. 1 Body cards are used to suggest simple movements.

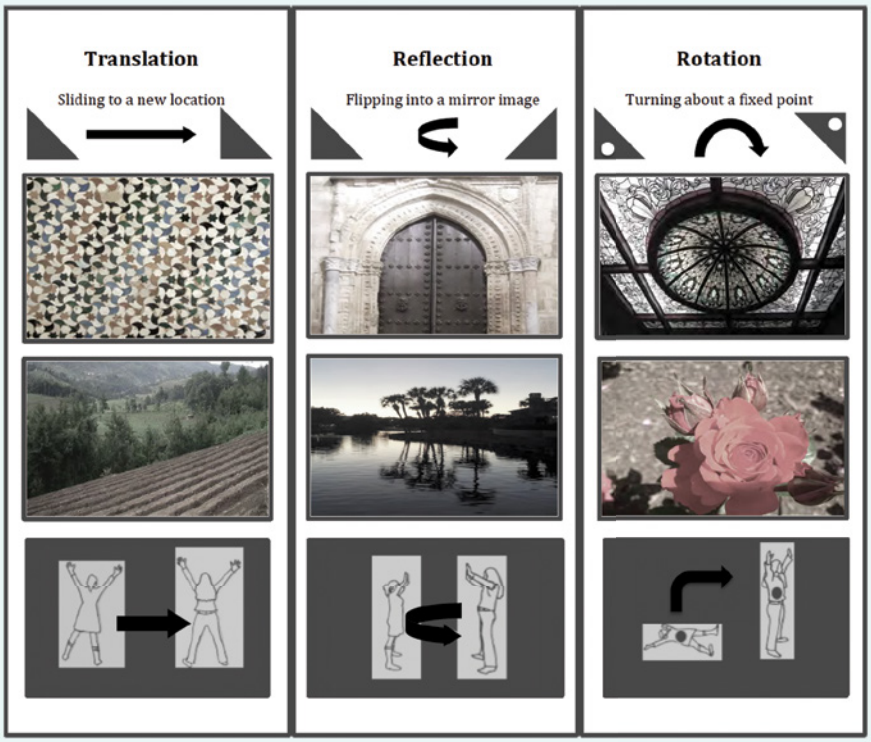
WARM-UP

The warm-up task takes about 5 to 8 minutes and involves a whole-group breathing exercise and simple physical warm-up based on geometry terminology and body movements used in the task. The goals for the warm-up are threefold: (a) Introduce students to essential mathematics vocabulary in context; (b) preassess students' conceptual understanding of isometric transformations; and (c) focus the energy of the group and ease into movement inquiry and embodied practices in a low-risk environment, which is particularly important if transitioning from more desk-oriented work. Moving in front of people may feel uncomfortable to some students—and even some teachers—at first. Therefore, a simple group warm-up provides a means to focus on the activity rather than individuals, which supports equitable participation in the context of students' varying comfort levels.

1. Gather students in a circle. Direct students as a group to take three deep breaths.
2. Lead students in simple stretching movements that use the body parts and body regions that will be used during the activity (see **fig. 1** for examples). For example, direct students to slowly roll their heads to the left and right and look up and down, just as they might when beginning a warm-up in physical education, at sports practice, or in a dance class. Also, direct students to make shapes, such as using their arms to make a triangle. Ask students to make a shape or stretch with body regions, such as the right or the left side of the body, so that each side is emphasized. Incorporate gestures into the stretching work, such as a shrug of the shoulders or a wave of the hand.
3. Expand the stretching activity to include movements based on isometric transformations. For example, ask students to transform a shape, pose, or gesture you perform, such as reflecting a "peek-a-boo" gesture with their hands, rotating their arms and shoulders to mimic a brushing-teeth motion, or translating a 45 degree angle made with a right leg by hopping to the right three times. Emphasize terms in context, and model possible interpretations for students as needed. Formatively assess students' spatial-thinking skills and initial understandings of transformations by observing how they embody reflections, rotations, and translations. You may want to make mental or written notes about students who need additional support during the group exploration task. However, we discourage verbally "correcting"

IMAGE ON PREVIOUS PAGE, SKYNESHER/THINKSTOCK; GIRL, GRADYREESE/THINKSTOCK

Fig. 2 Infographics show six representations for each of the isometric transformations.



student movements during the warm-up task because students will have opportunities to revise their thinking during the small-group exploration. Keep in mind that there may be multiple ways for students to accurately embody these transformations.

We have found that some students demonstrate discomfort by laughing, particularly if this type of activity is atypical for their math classes. However, practices like the ones described will become routine with frequent use. What is more, a warm-up of this type can be used to calm and focus students before a test or productively redirect frenetic classroom energy.

EXPLORATION

The two goals for the exploration task, which takes about 15 to 20 minutes, are (a) to uncover students' prior knowledge about transformations and (b) to introduce students to body movements through nonthreatening, open-ended opportunities for sorting and classifying. We recommend that teachers prepare a transformations infographic for each group in advance (see **fig. 2**), as well as make a set of 12 "body cards" that depict simple movements that can be performed or initiated by the head, shoulders, torso, arms, elbows, hands, hips, legs, feet, right side, left side, and whole body (see **fig. 1** for examples). These cards serve as prompts for movement creation and need not be prescriptive or limiting.

1. Assemble students into small groups and tell them to sit in a circle on the floor. We recommend groups of about four and encourage random group assignment to communicate the teacher's high expectations for competence and participation for every student. However, we support adaptations to this strategy on the basis of the teacher's knowledge of his or her students. In this case, it is critical to avoid sorting students into groups based on perceptions of students' mathematical or physical abilities because of the potential for unintentionally amplifying existing status problems and devaluing contributions from lower-status students (Cohen and Lotan 2014).

2. Tell students to think silently on their own for 30 seconds about what they already know about reflections, rotations, and translations. Then direct students to go round-robin and share one idea or example at a time in their groups. Without joining any of the groups, observe and listen to the ideas across groups, and make notes about what students do and do not understand. Keep this discussion brief to prevent losing momentum.

3. Distribute a copy of the transformation infographic to each group (see **fig. 2**), which contains six representations for each of the three isometric transformations (e.g., words, definitions, shapes, human-made structural forms, in nature or agriculture, and as human-embodied shapes). Direct groups to make sense of the representations and brainstorm additional examples that would work. Time permitting, ask student groups to create their own infographic on poster paper using hand-drawn images of the examples they generated. As before, observe and listen to student thinking, and make notes about what students do and do not understand. To prevent student groups from relying on the teacher as the source and arbiter of knowledge for this task, intervene only when necessary.

4. Distribute a set of body cards to each group when they are ready

to move on (see **fig. 1**). Encourage groups to try out the movements on the cards and experiment with their own movements or gestures. Keep in mind that movements can range from simple gestures to more stereotypical dance moves. Students will have opportunities to develop their ideas in the composition task, so move groups on as soon as students are familiar with the cards.

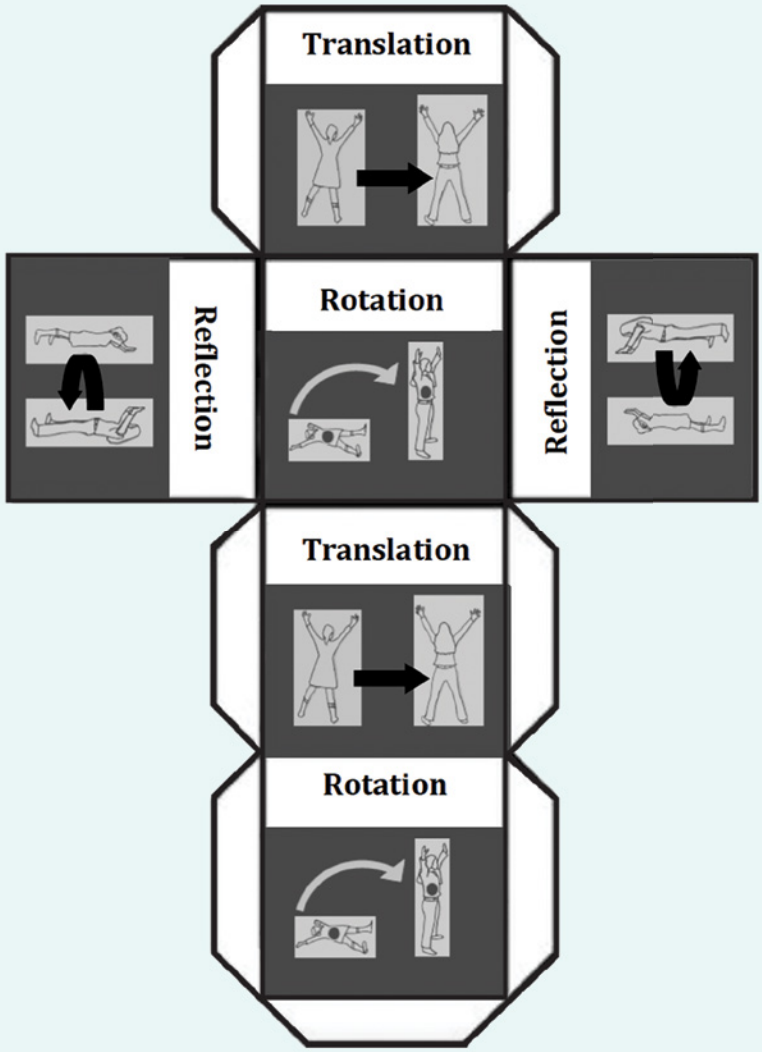
5. Direct groups to organize the body cards on the floor in a way that makes sense them (e.g., see **figs. 3a** and **3b**). Although most groups will organize the cards in relationship to where they are located on the body, some groups may base their strategy on the shape of a person or on larger body region clusters (e.g., top half, bottom half, sides). Ask groups to walk around together and make sense of the other groups' organization schemes.

We have found that a brief whole-group discussion at this point is important for addressing students' questions, synthesizing where we are in the task, and providing a transition point for the composition task. We have found that using well-documented talk moves and questioning techniques helps us facilitate the discussion in ways that support student interdependence and autonomy (Boaler and Humphreys 2005; Chapin, O'Connor, and Anderson 2013). We begin the discussion by prompting student groups to ask each other clarifying questions about the infographic they created (optional) or their organization scheme for the body cards. We transition to the next task by asking students to think about what it might mean to embody reflections, rotations, and translations of their movements.

Fig. 3 Students organize the body cards on the floor in any way that makes sense to them.

(a) (b)

Fig. 4 Each student rolls the die to determine the transformation to be embodied.



COMPOSITION

The composition task, which takes about 15 to 20 minutes, challenges student groups to choreograph a dance sequence using transformations-based movements. Composing sequences of dance movements that accurately embody transformations is no easy task, as it requires grappling with what reflections, rotations, and translations mean in relation to space. The goals of the composition task are to (a) create transformations dance movements by applying transformations to an original movement or gesture, (b) sequence these movements together smoothly into a dance sequence, and (c) create an important reference point for defining congruence and similarity as sequences of transformations later on. Each group needs a set of body cards and one hand-made six-sided die with transformations printed on them (see **fig. 4**).

1. Direct groups to stand in a circle and shuffle their set of body cards and place them face down in a pile on the floor. Distribute dice to groups (1 per group). Announce that every student will get at least one turn.

2. A turn consists of one student picking a body card and then creating and performing a simple movement or gesture that uses the body part or region depicted on the card. Students need not include technical dance-like movements, as there is no one right way to dance in this task. The person on the student's left will roll the die and announce the transformation that was rolled. Then the entire student group practices embodying possible applications of the transformation to the original movement, which completes the turn. This process is repeated by moving clockwise around the group so that every student has the opportunity to pick a card and create a movement, and every student has a chance to roll the die. We recommend that student groups complete two full rounds so that each student creates at least two movements and two transformations, although teachers may need to adapt this process based on their knowledge of their own students.

3. Direct groups to sequence the movements they created into one combined dance sequence. Students should include moves that help them smoothly transition from one movement to the next. For example, if one movement is created while standing and the next person creates a movement while sitting, choreographing a transition movement in between or creating a movement that fluidly transitions from standing to seated is optimal. Assess student progress by observing and listening. What can you tell about their understanding of isometric transformations just by observing them? What questions do they ask one another?

4. Tell student groups to invite you to join them for a check-in once they complete one pass through their dance sequence. Ask them to perform one of the dance moves in their sequence and explain how they incorporated transformations into this move. Press their thinking with probing questions, such as asking whether a particular move is a reflection or a rotation and whether it can be both. Then prepare students for a successful performance experience in the next task by asking whether they are comfortable sharing their dance with the class. Tell students what to expect so that they can ease into the idea of physically sharing their ideas. Group members who are uncomfortable with performance do not have to do it, as they will still benefit from the other aspects of this embodied learning experience.

5. Tell student groups to continue rehearsing their dance sequence in preparation for performing it for the class. Determine performance order for the next task on the basis of feedback from group check-ins, and share this information with groups. Depending on time and logistics, teachers may allow solos, duets, unison group performance, or combinations thereof as an extension task or homework assignment.

Although students will likely be comfortable creating and performing movements in the cultivated safety of their small groups, this task invites students' individual contributions of embodied movements to be first shared only with their groups. Although we strongly endorse giving students the chance to encourage and support one another, teachers should be on the lookout for students who may become distressed and intervene where necessary. For example, in the unlikely event that a student is too overwhelmed by being in the spotlight, teachers might suggest creating hand gestures from a desk or seated position. Sometimes the best intervention is giving students the space they need to calm down or suggesting they take a quick water break and rejoin the group when they are ready.

It is important to keep in mind that this task is inclusive by design, as any movements that students are

capable of creating are valid and valued. Students with mobility devices or other physical aids are especially valuable members for their groups, as dance moves built around these tools inspire especially creative, original dance moves. Other group members or paraprofessional aides are welcome to assist by pushing a student's wheelchair, for example, or acting as a safety spotter.

SHARING

The sharing task, which takes about 10 to 15 minutes, involves student performance of their dance sequences and peer feedback. We encourage a presentation in which each group performs its dance in front of the whole class, although we recognize that some teachers may adapt this task for the small-group setting and observe student groups accordingly. Either way, each group should perform their dance at least twice—the first time as they rehearsed it and then the second time while talking through the mathematics as they perform (see the video clip, for example, which is found with the more4U at http://www.nctm.org/mtms). Groups are responsible for explaining how they applied transformations to the dance movements they sequenced together for their choreographed composition. Audience members should be told what counts as successful audience participation, such as giving peer encouragement, observing respectfully, avoiding put-downs and talking during the performance, and providing feedback according to the teacher's preferred protocol. For example, teachers may use the "T.A.G" protocol to support productive audience member participation—tell something that you noticed; ask a question; give an affirmation or suggestion. Audience accountability can be supported by the inclusion of written feedback, such as by providing several small slips of paper with the protocol printed on it and a space where the students can write their name. Feedback can be shared verbally or in written form, which allows the teacher to review the comments before sharing them with the groups.

IMAGE BY GRADYREESE/ISTOCK

We encourage a presentation in which each group performs its dance in front of the whole class.

REFLECTION

The purpose of the reflection task, which takes about 5 minutes, is to synthesize the content ideas addressed in the lesson and acknowledge the closure of the performance task before transitioning to other work. It may not be easy for students to transition from an embodied movement activity to one that requires calm and stillness, such as problem-solving tasks or individual seatwork.

1. Gather students in a circle. Direct students to take three deep breaths as a group.
2. Tell students to continue taking three more deep breaths and, as they do so, invite them to think silently about the contributions they made to their groups, as well as any challenges they faced during this task.

Conclude the performance task by asking students to complete an individual reflection exercise, such as a journal entry in which they define reflections, rotations, and translations; describe what they learned that they did not already know; celebrate contributions they made to their groups; and write about the challenges they faced during the *Transformations Dance*. Teachers may also prompt students to solve one or two transformations problems to assess how students initially apply the conceptual and spatial reasoning skills they developed in this task to traditional problem-solving scenarios.

SPATIAL REASONING THROUGH MOVEMENT

The Transformations Dance has proven to be an important movement-as-inquiry approach for the preservice and in-service middle school mathematics teachers we work with, as well as for their students' learning of early transformational geometry. By engaging in this performance task, students have multiple opportunities to develop their spatial-reasoning skills and advance

their conceptual understanding of reflections, rotations, and translations. This work prepares them to understand congruence and similarity as sequences of transformations, which can be difficult for many students because of the spatial fluency required to make sense of these ideas. By having students choreograph dance sequences based on their embodied representations of transformations, this performance task provides an adaptable strategy for engaging early adolescents in high-priority content in ways that also support their growth as developing adolescents.

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Alison E. Leonard, aleona2@clemson.edu, is an assistant professor of arts and creativity in the College of Education at Clemson University, Clemson, South Carolina. She teaches an arts education (dance, drama, music, and visual arts) course for preservice teachers and is particularly interested in embodied forms of inquiry and dance in schools. **Nicole A. Bannister**, nbannis@clemson.edu, is an assistant professor of mathematics education in the College of Education and College of Science at Clemson University. She teaches courses for middle school and secondary preservice teachers and is interested in collaborative teacher learning and equitable praxis.